Graphic Equalizer Design Using Higher-Order Recursive Filters

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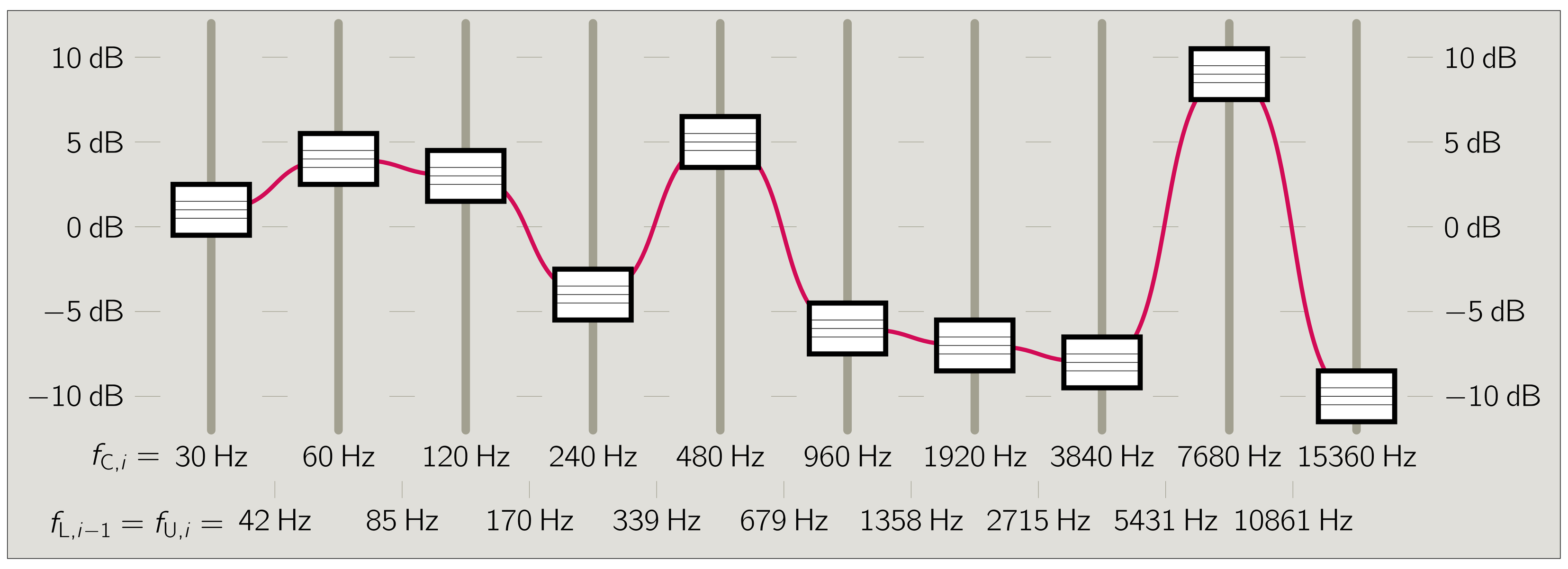
Graphic Equalizers



- control gain in a number of adjacent bands
- sliders' knobs represent samples of the frequency response
- band center frequencies distributed logarithmically:

$$f_{C,i} = R \cdot f_{C,i-1}, \quad R = \begin{cases} 2 & \text{for octave equalizer} \\ \sqrt[3]{2} & \text{for 1/3-octave equalizer} \end{cases}$$

- band edges: geometric mean of center frequencies

$$f_{L,i} = f_{C,i}/\sqrt{R}, \quad f_{U,i} = f_{C,i} \cdot \sqrt{R} = f_{L,i-1}$$

- bandwidth:

$$f_{B,i} = f_{U,i} - f_{L,i} = f_{C,i} \cdot \left(\sqrt{R} - \frac{1}{\sqrt{R}}\right)$$

Band Shelving Filters

Desired properties

- easy specification of the gain in the band

$$\left|H(e^{j\Omega_C})\right| = g$$

- near unity gain outside the band

$$\left|H(e^{j\Omega})\right| \approx 1, \ \Omega \notin [\Omega_L, \Omega_U]$$

- complementary filter edges, especially half the gain in dB at the edge frequencies

$$\left|H(e^{j\Omega_L})\right| = \left|H(e^{j\Omega_U})\right| = \sqrt{g}$$

Analog low-shelving prototype

$$H_{LS,A}(s) = \prod_{m=1}^{M} \frac{s + e^{j\alpha_m}\sqrt[M]{g}}{s + e^{j\alpha_m}}$$

$$\alpha_m = \left(\tfrac{1}{2} - \tfrac{2m-1}{2M}\right)\pi$$

Digital low-shelving prototype

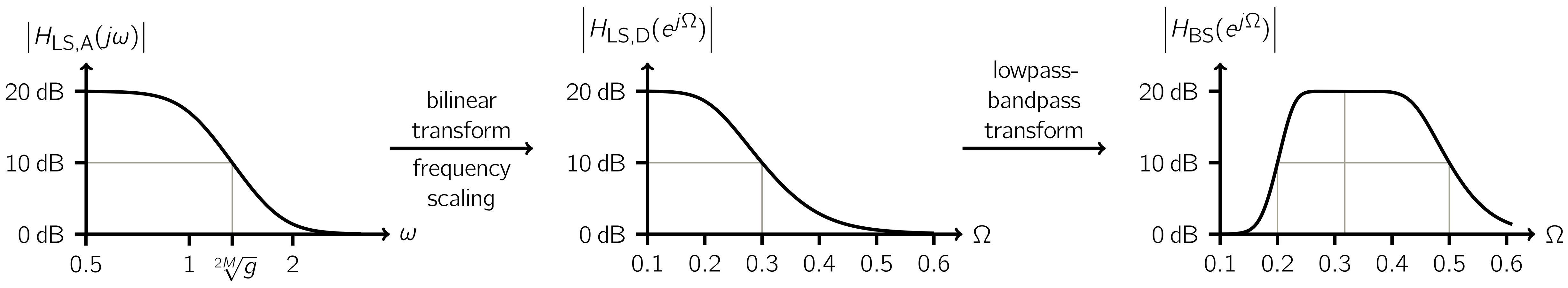
$$H_{LS,D}(z) = H_{LS,A}\left(\frac{1}{K} \cdot \frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$K = \frac{1}{\sqrt[2M]{g}} \tan\left(\frac{\Omega_B}{2}\right)$$

Digital band-shelving filter

$$H_{BS}(z) = H_{LS,D}\left(z\frac{\cos\Omega_M - z}{1 - \cos\Omega_M z}\right)$$

$$\tan^2\left(\frac{\Omega_M}{2}\right) = \tan\left(\frac{\Omega_L}{2}\right)\tan\left(\frac{\Omega_U}{2}\right)$$

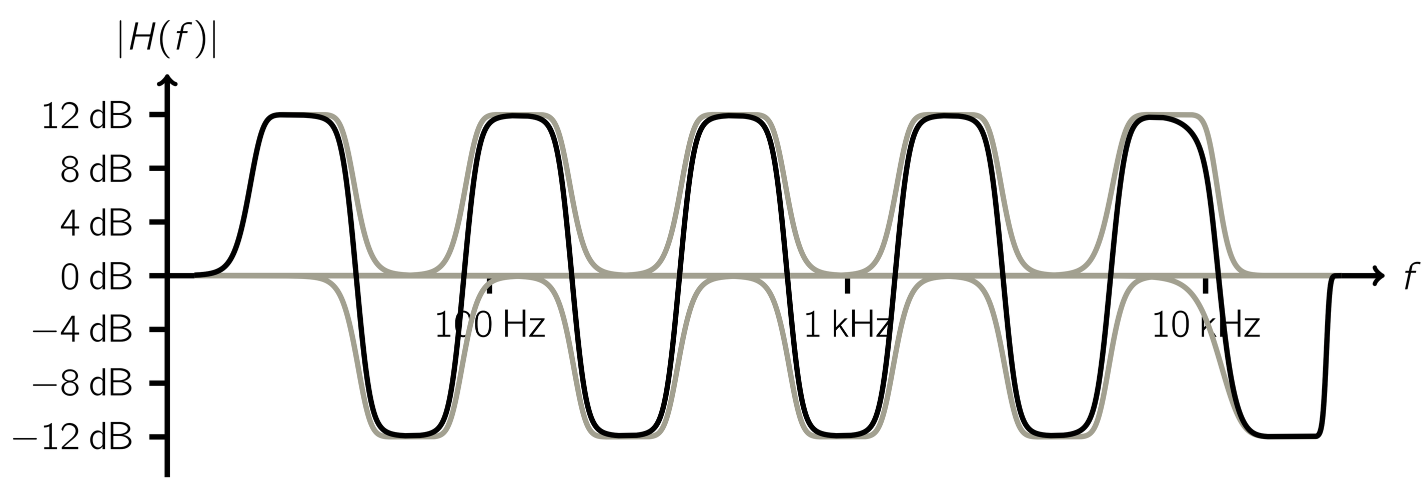


$g = 10$, filter order $M = 4$ $\quad$ $\Omega_B = 0.3$ $\quad$ $\Omega_L = 0.2, \quad \Omega_U = 0.5$

Example octave equalizer

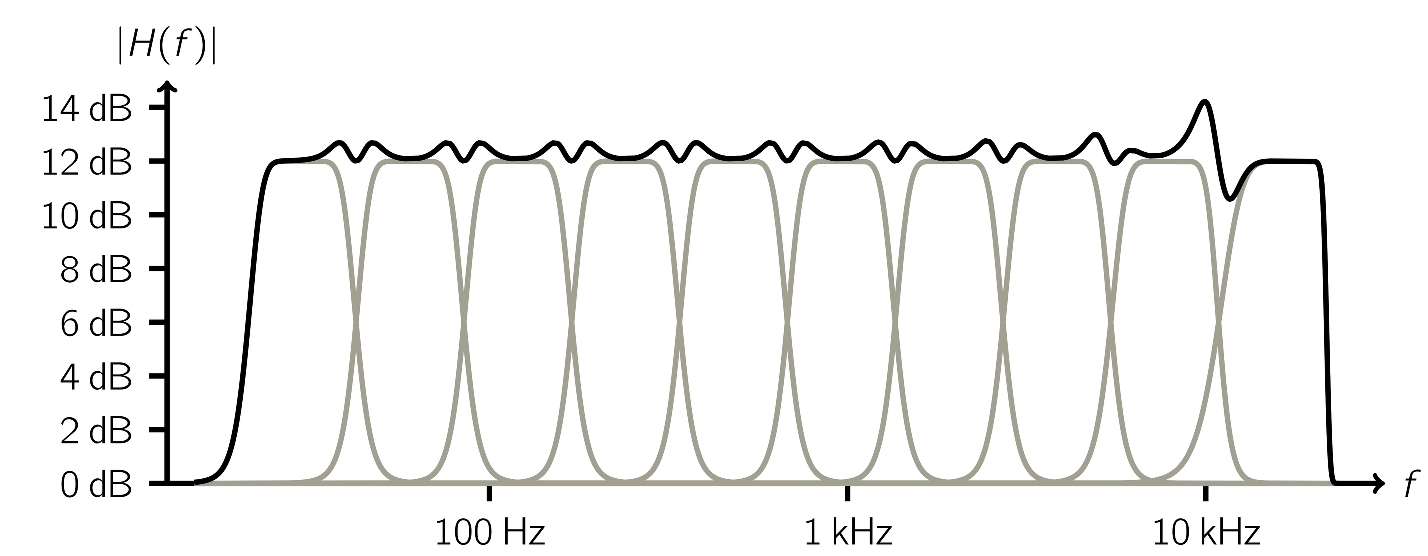
Order of band-shelving filters: 8 ($M = 4$)

Different gain in adjacent bands



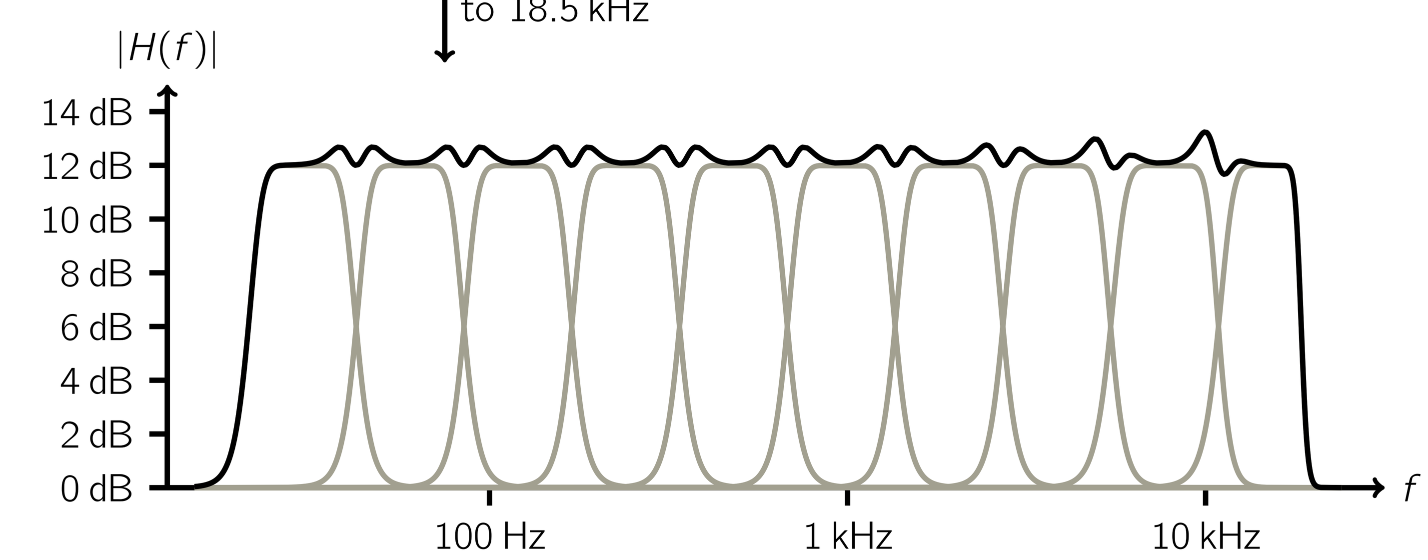
- no significant influence across bands

Same gain in all bands



- nearly constant amplitude, except for transitional region below highest band

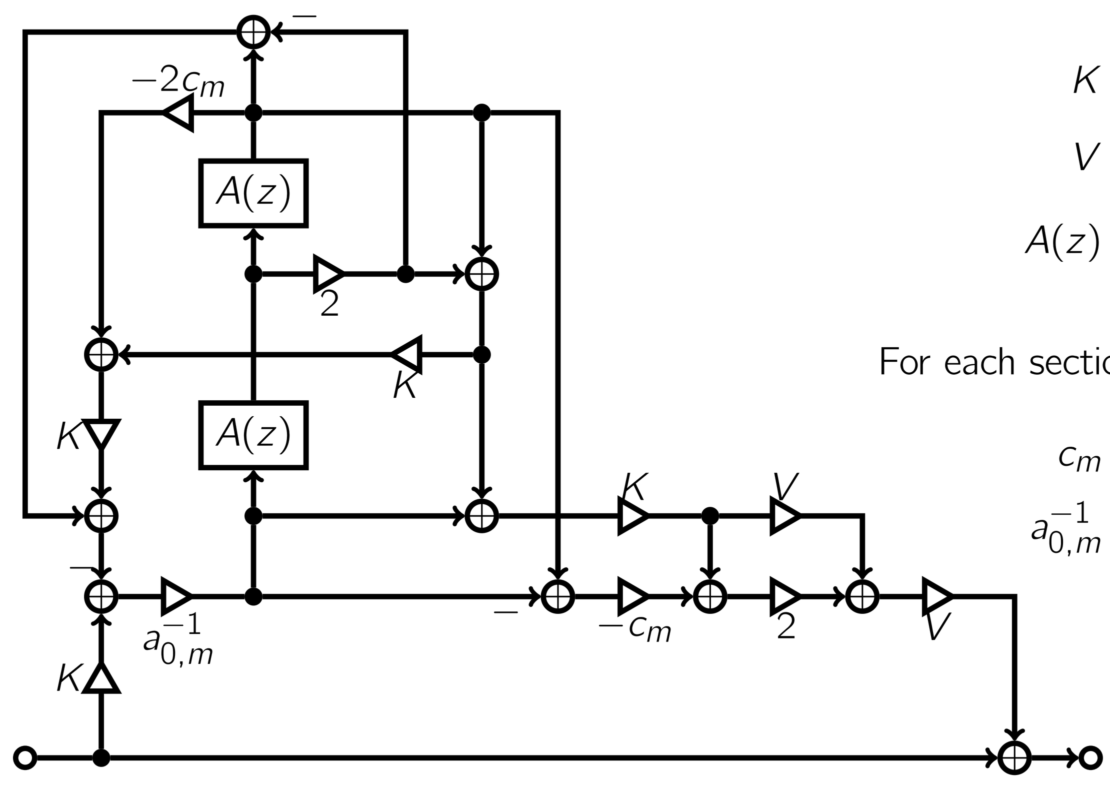
Reduce f_U of highest band from 21.7 kHz to 18.5 kHz



- nearly constant amplitude across whole frequency axis

Implementation

Realization of the shelving filters as a cascade fourth-order sections



For each band:

$$K = \frac{1}{\sqrt[2M]{g}} \tan\left(\frac{\Omega_B}{2}\right)$$

$$V = \sqrt[M]{g}$$

$$A(z) = z^{-1}\frac{\cos\Omega_M - z^{-1}}{1 - \cos\Omega_M \cdot z^{-1}}$$

For each section ($m = 1, \ldots, \frac{M}{2}$) in each band:

$$c_m = \cos(\alpha_m)$$

$$a_{0,m}^{-1} = \frac{1}{1 + 2Kc_m + K^2}.$$

Java demo applet

http://ant.hsu-hh.de/jdafx