A New Analysis Method for Sinusoids+Noise Spectral Models

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Introduction

- We consider stationary Sinusoids+Noise sounds.
- We can decompose such a sound into:
  - deterministic part
    - sum of sinusoids evolving slowly
    - characterized by phase, frequency, amplitude;
  - stochastic part
    - filtered white noise
    - characterized by its spectral envelope (color).

Usual Determination of the Stochastic Part

1. classification of the spectral peaks (sinusoids or noise);
2. extraction of the sinusoidal peak parameters (Φ, f, a);
3. re-synthesis of the deterministic (sinusoidal) part;
4. subtraction to obtain the residual;
5. smoothing of the residual spectrum.

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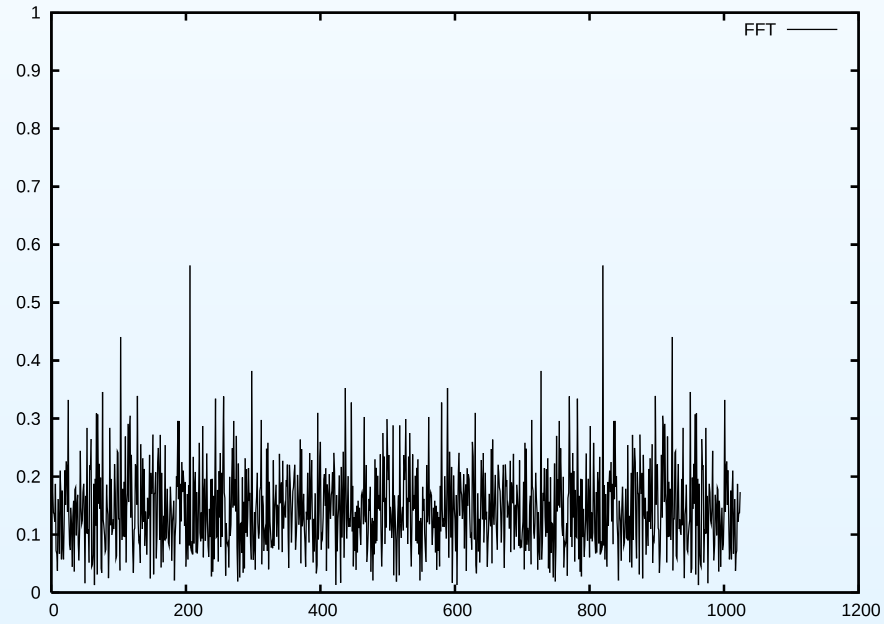
Limitations:

- classification is a binary decision;
- cannot avoid estimation errors in sinusoidal parameters on noisy bins (Cramer-Rao Bound);
- errors in the sinusoidal parameters lead to errors in the residual. . .

Proposed Approach

- We consider magnitude variations along the time axis.
- In a stationary sound
  - high variations indicates the presence of noise;
  - stationarity characterizes sinusoidal components.



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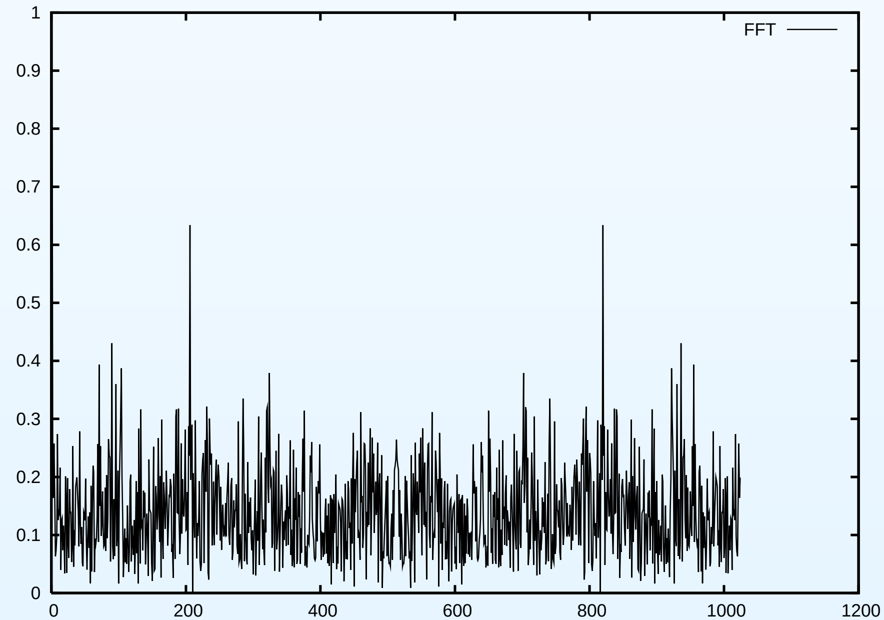

- Our method relies on statistical magnitude variations.

Distribution of the Magnitude Spectrum: Noise

- Thermal Noise

$$X(t) = \sum_{n=1}^{N} A_n \cos(\omega_n t) + B_n \sin(\omega_n t)$$

- The magnitude on bin n is defined by random variables C_n:

$$C_n = \sqrt{A_n^2 + B_n^2}$$

- The magnitude distribution of a stationary noise follows a Rayleigh PDF.

Distribution of the Magnitude Spectrum: Sines+Noise

- The magnitude on a noisy bin occupied by a sinusoid of magnitude A is defined by random variables M:

$$M = \sqrt{(N_r + A_r)^2 + (N_i + A_i)^2}$$

- M follows a Rice PDF.

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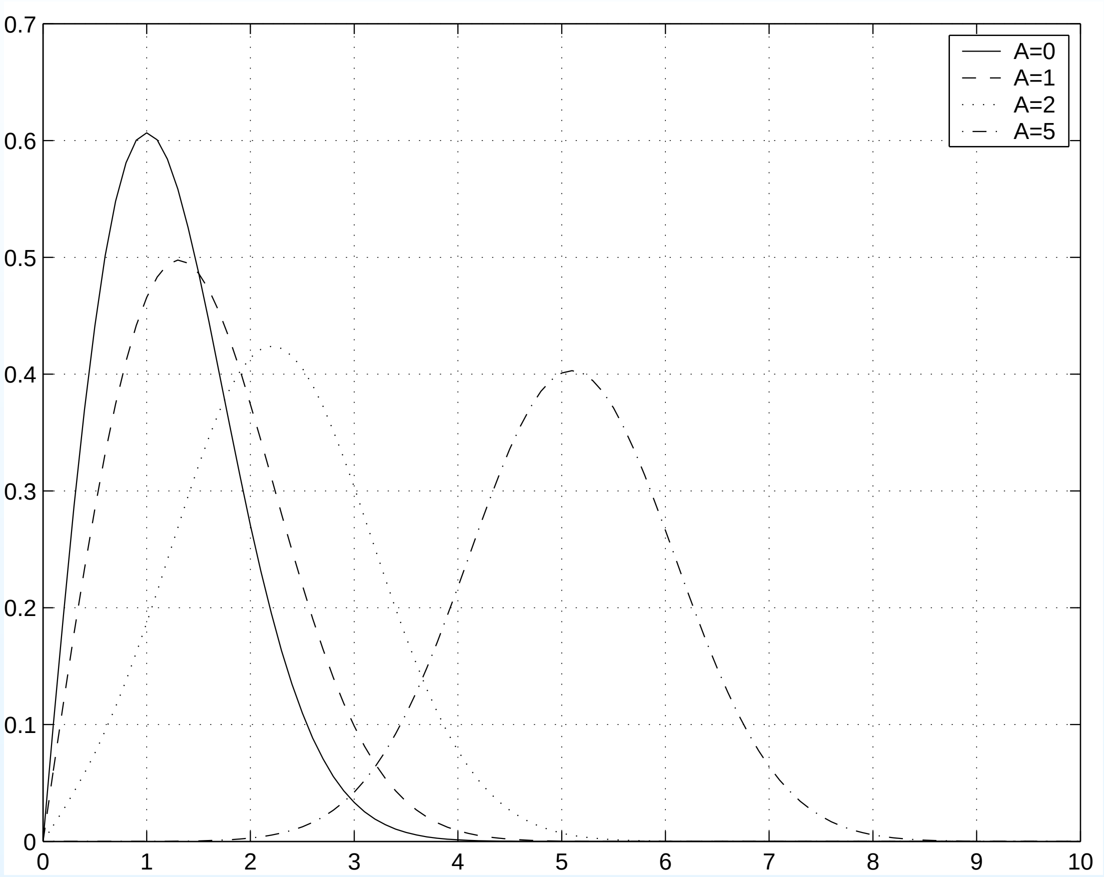
Rayleigh and Rice PDF



Rice Probability Distribution Function.

Rayleigh and Rice PDF

- Rayleigh PDF:

$$p\left(M_i\right) = \frac{M_i}{\sigma^2} e^{\frac{-M_i^2}{2\sigma^2}}$$

- Rice PDF:

$$p_{A,\sigma}\left(M_i\right) = \frac{M_i}{\sigma^2} e^{\frac{-\left(M_i^2 + A^2\right)}{2\sigma^2}} I_0\left(\frac{AM_i}{\sigma^2}\right)$$

where $\quad I_0\left(x\right) = \frac{1}{2\pi}\int_0^{2\pi} e^{x\cos(\phi)}\, d\phi$

- when $A = 0$, the Rice PDF turns into the Rayleigh PDF.

Noise Estimation from Magnitude Distribution

- The Rice PDF is a 2-parameter function (A, σ).
- For each bin, the complex spectrum can be characterized by:
  - the parameter A that represents
    the magnitude from a sinusoidal peak,
  - the standard deviation σ that represents
    the noise energy on this bin.

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Noise Estimation from Magnitude Distribution

- We consider the magnitude distribution on a single bin on consecutive frames.
- The noise power density can be obtained by the estimation of σ on each bin.
- Since this distribution is defined by the Rice PDF, we can use Rice parameters estimators:
  - Moments based method
  - Maximum likelihood method

Rice Parameters Estimator: Moments

- First moment:

$$E\left[M\right] = \sigma\sqrt{\frac{\pi}{2}}e^{\frac{-A^2}{4\sigma^2}}\left[\left(1+\frac{A^2}{2\sigma^2}\right)I_0\left(\frac{A^2}{4\sigma^2}\right)+\left(\frac{A^2}{2\sigma^2}\right)I_1\left(\frac{A^2}{4\sigma^2}\right)\right]$$

- Second moment:

$$E\left[M^2\right] = A^2+\sigma^2$$

- A and σ obtained by finding the pair that matches the calculated moments.

Rice Parameters Estimator: Maximum Likelihood

- Log-likelihood function:

$$\log(\mathcal{L}) = \sum_{i=1}^{N} \frac{M_i}{\sigma^2} I_0\left(\frac{AM_i}{2\sigma^2}\right) - \frac{NA^2}{2\sigma^2} - \sum_{i=1}^{N} \frac{M_i^2}{2\sigma^2}$$

- We search the pair (A,σ) that maximizes the log-likelihood.

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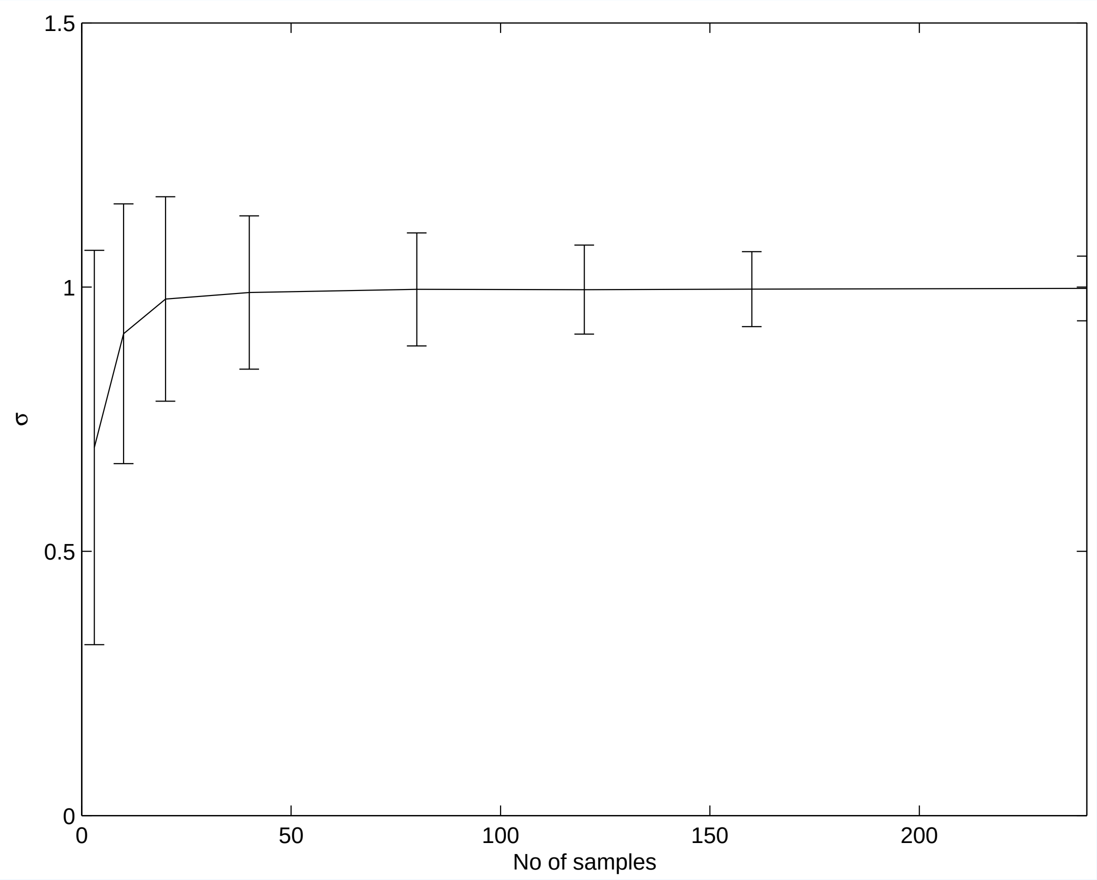
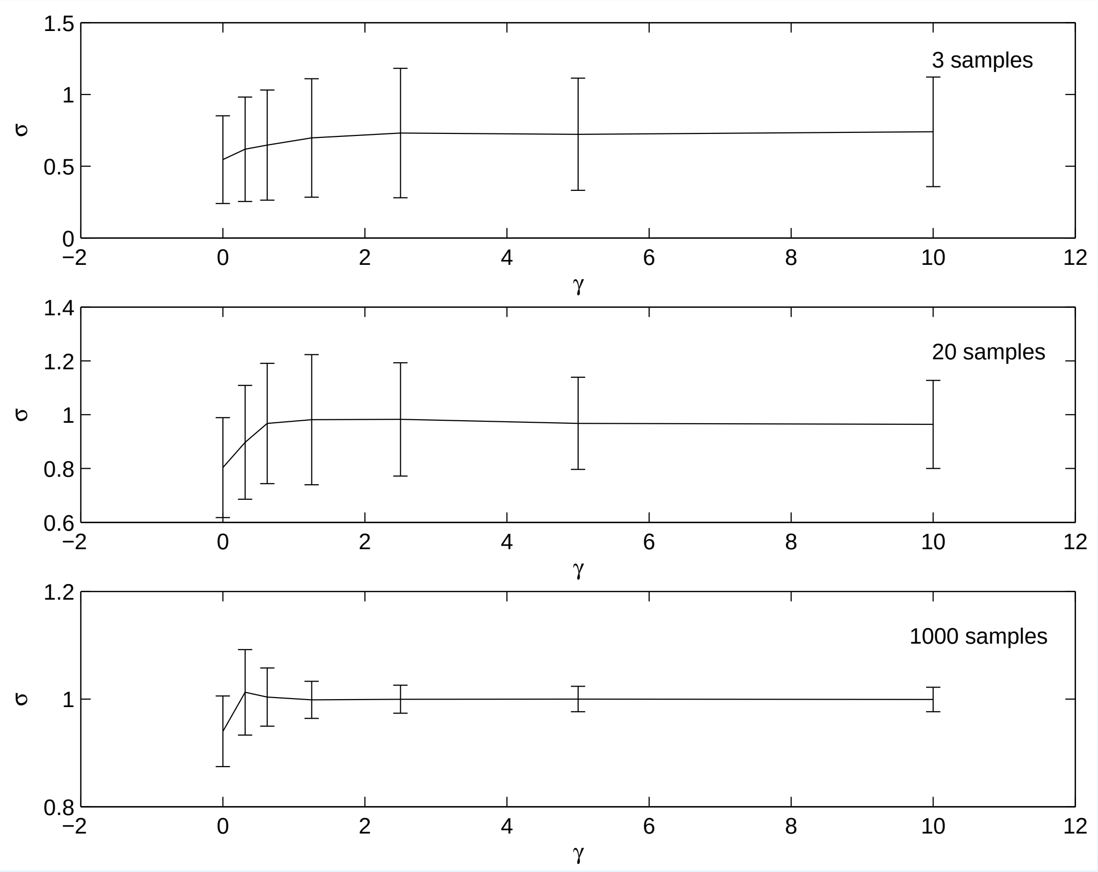
Influence of the Number of Samples



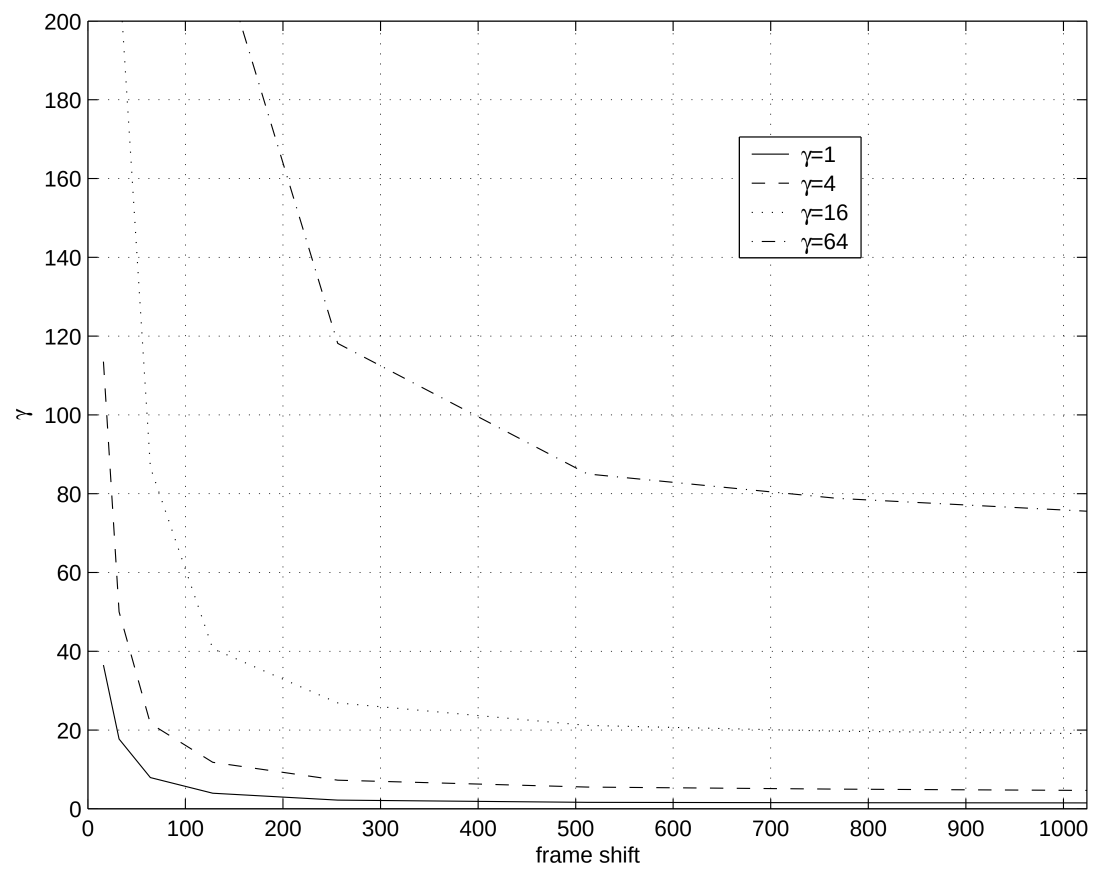
Mean and standard deviation as functions of the number of samples.

Influence of the SNR



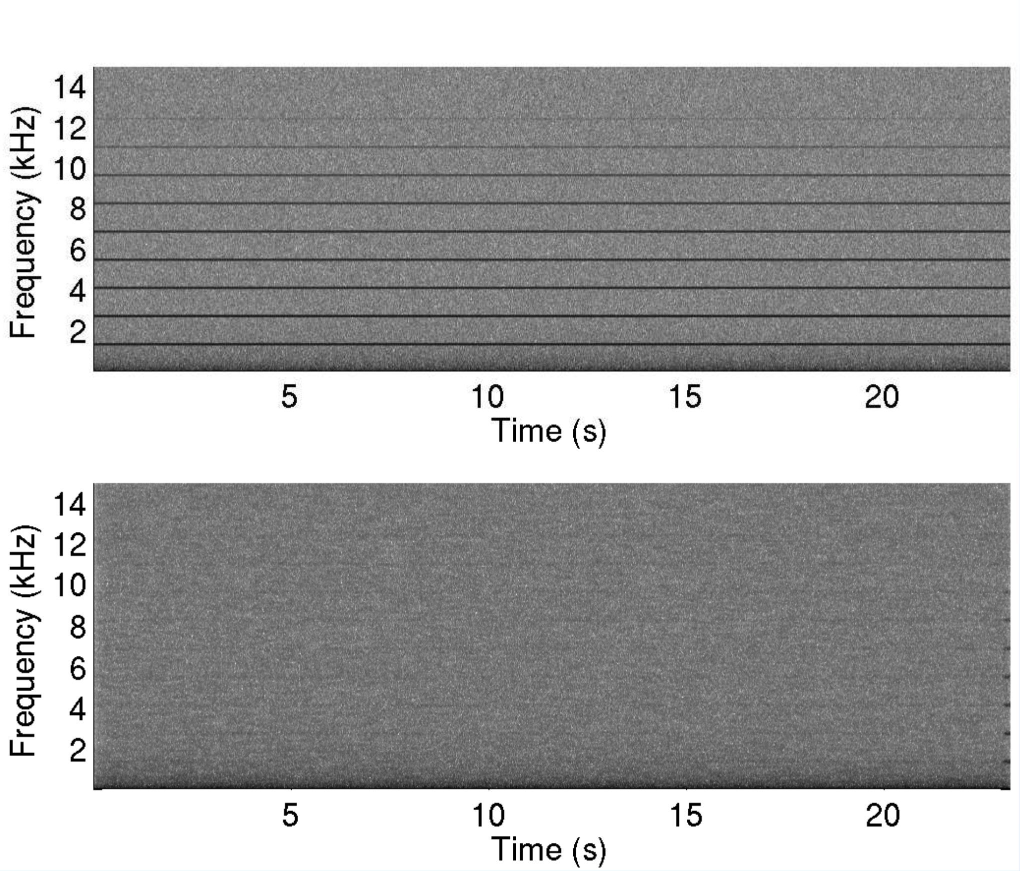
Mean and standard deviation as functions of SNR for various numbers of samples.

Influence of the Overlap



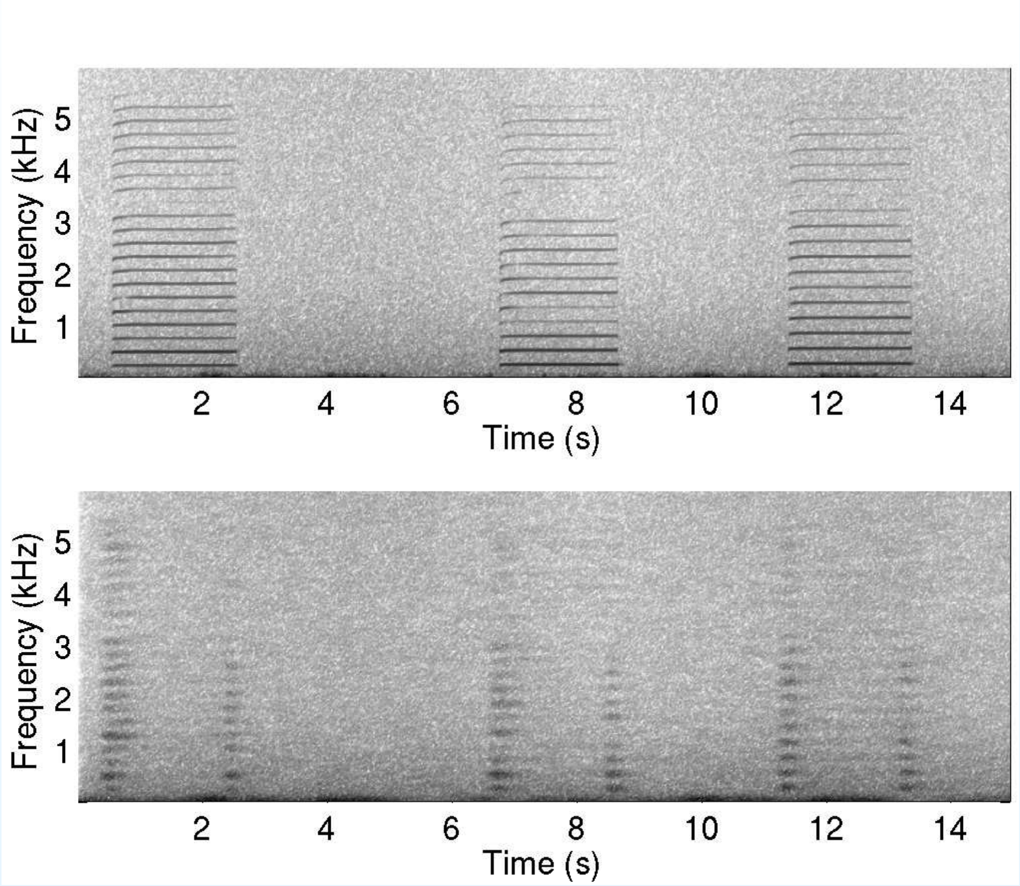
Estimated SNR (γ) as a function of the overlap shift.

Sound Examples: Synthetic



- synthetic sound – original
- synthetic sound – resynthesis

Sound Examples: Natural



- saxophone + wind – original
- saxophone + wind – resynthesis

Conclusion

- This method relies on a long-term analysis of the variations of the amplitude spectrum.
- We estimate the stochastic part of the signal without having previously estimated the deterministic part.
- We avoid errors due to the estimation of the sinusoidal parameters for the noisy bins.